Mathematical Economics
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SAMPLE PROBLEMS RECOMMENDED FOR FINAL PREPARATION.

USE OF THE DERIVATIVE IN ECONOMICS

1) A firm has the demand function $22 - 0.5Q - P = 0$ and the average cost function

$$AC = \tfrac{1}{3}Q^2 - 8.5Q + 50 + 90/Q.$$

Find the level of output which maximizes (*a*) total revenue and (*b*) total profits.

CONSTRAINED OPTIMIZATION IN ECONOMICS

(a) What combination of goods x and y should a firm produce to minimize costs when the joint cost function is $c = 6x^2 + 10y^2 - xy + 30$ and the firm has a production quota of $x + y = 34$? (b) Estimate the effect on costs if the production quota is reduced by one unit.

(a) What output mix should a profit-maximizing firm produce when its total profit function is $\pi = 80x - 2x^2 - xy - 3y^2 + 100y$ and its maximum output capacity is $x + y = 12$? (b) Estimate the effect on profits if output capacity is expanded by one unit.

A rancher faces the profit function

$$\pi = 110x - 3x^2 - 2xy - 2y^2 + 140y$$

where x = sides of beef and y = hides. Since there are two sides of beef for every hide, it follows that output must be in the proportion

$$\frac{x}{2} = y \qquad x = 2y$$

At what level of output will the rancher maximize profits?

max π wrt x and y

CONSTRAINED OPTIMIZATION OF A GENERALIZED COBB–DOUGLAS FUNCTION

7) Given a production function $q = K^{0.4}L^{0.5}$ subject to a budget constraint of \$108 when $P_K = 3$ and $P_L = 4$, the Lagrange function for optimization is

$$Q = K^{0.4}L^{0.5} + \lambda(3K + 4L - 108)$$

8) Given a production function $q = K^{0.3}L^{0.5}$ subject to the constraint $6K + 2L = 384$, find the maximum output subject to the constraint.

9) (*a*) Minimize costs for a firm with the cost function $c = 5x^2 + 2xy + 3y^2 + 800$ subject to the production quota $x + y = 39$. (*b*) Estimate additional costs if the production quota is increased to 40.

10) A monopolistic firm has the following demand functions for each of its products x and y:

$$x = 72 - 0.5P_x \tag{6.46}$$

$$y = 120 - P_y \tag{6.47}$$

The combined cost function is $c = x^2 + xy + y^2 + 35$ and maximum joint production is 40. Thus, $x + y = 40$. Find the profit-maximizing level of (*a*) output, (*b*) price, and (*c*) profit.

(*a*) Maximize utility $u = Q_1Q_2$, when $P_1 = 1$, $P_2 = 4$, and one's budget, $B = 120$.
(*b*) Estimate the effect of a one-unit increase in the budget.

(*a*) Maximize utility $u = Q_1Q_2$, subject to $P_1 = 10$, $P_2 = 2$, and $B = 240$. (*b*) What is the marginal utility of money?

Maximize utility $u = Q_1Q_2 + Q_1 + 2Q_2$, subject to $P_1 = 2$, $P_2 = 5$, and $B = 51$.